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MODEL OF HEAT TRANSFER IN BUBBLE BOILING

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Based on representation of the bubble-boiling process as near-wall turbulence, limiting relations are obtained that describe heat transfer for the low- and high-pressure regions.

Introduction. The first theoretical representations in the field of bubble boiling belong to Labuntsov [1]. In [2], he obtained fundamental relations for the growth rate of a vapor bubble on a wall and the heat-transfer coefficient in boiling. According to the approach developed in [2] the specific heat flux q transferred from a heated wall to a boiling liquid is a sum of two components

$$q = q_1 + q_2 . \tag{1}$$

The component q_1 is the heat transferred by conduction through a viscous sublayer of thickness δ:

$$q_1 = \lambda \Delta T / \delta .$$

The quantity δ is calculated by analogy with one-phase forced convection as [3]

$$\delta = \nu / W ,$$

where $W = R(dR/dt)\sqrt{N}$ is the dynamic velocity corresponding to "the effective turbulence of boiling." Hence the first term in (1), which describes the heat transfer in the low-pressure region, is obtained:

$$q_1 \approx \frac{\lambda^2 \Delta T^3}{\nu \sigma T_s} . \tag{2}$$

The component q_2 is the heat transferred by liquid evaporation involving formation of vapor bubbles:

$$q_2 = r \rho'' W .$$

This gives the second term in (1), which describes the heat transfer in the high-pressure region:

$$q_2 \approx \frac{\lambda r \rho'' \Delta T^2}{\sigma T_s} . \tag{3}$$

The next step in development of the theory of bubble boiling was made by Yagov [4], who pointed to the decisive role of "dry spots" on a high-pressure heating surface. In [4] a dependence $q(\Delta T)$ is obtained that asymptotically passes over to relation (2) in the low-pressure range. The asymptotic form of the solution [4] in the high-pressure range is as follows:

$$q_2 \approx \frac{\nu^{1/2} r \Delta T^3}{\sigma^2} \left(\frac{\lambda \rho''}{T_s} \right)^{3/2} . \tag{4}$$

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Thus, for the low-pressure region both theories, i.e., [2] and [4], give formula (2). For the high-pressure region formula (4) is preferable since it fits experimental data better. Passing from the coordinates $q(\Delta T)$ to $\alpha(q)$ in relations (2), (4), we arrive at the following limiting expressions for the heat-transfer coefficient:

$$\alpha_1 \approx \left(\frac{\lambda^2 q^2}{\nu \sigma T_s}\right)^{1/3} \tag{5}$$

for the low-pressure region;

$$\alpha_2 \approx \nu^{1/6} r^{1/3} \left(\frac{q}{\sigma}\right)^{2/3} \left(\frac{\lambda \rho''}{T_s}\right)^{1/2} \tag{6}$$

for the high-pressure region.

Recently experimental studies of the heat transfer in boiling of organic fluids and cooling agents whose results are generalized in a handbook on heat transfer published in Germany have been carried out [5]. The dependence of the heat-transfer coefficient on the specific heat flux is represented in [5] by the relation

$$\tilde{\alpha} = F \tilde{l}^{0.133} \tilde{q}^{m} . \tag{7}$$

Here $\tilde{\alpha} = \alpha/\alpha_0$, $\tilde{q} = q/q_0$, $\tilde{l} = l/l_0$ are the dimensionless values of the heat-transfer coefficient, the specific heat flux, and the characteristic dimension of roughness, respectively. The scales of the quantities in (7) are equal to

$$\alpha_0 = 0.1 \left(\frac{\lambda \rho r q_0^2}{\sigma T_s^2}\right)^{1/3} ; \quad q_0 = 2 \cdot 10^4 \ \mathrm{W/m^2} ; \quad l_0 = 4 \cdot 10^{-7} \ \mathrm{m} . \tag{8}$$

The quantities F and m in (7) are functions of the reduced pressure $\tilde{P}$:

$$m(\tilde{P}) = 0.9 - 0.3 \tilde{P}^{0.3} , \tag{9}$$

$$F(\tilde{P}) = 1.2 \tilde{P}^{0.27} + \tilde{P} \left(2.5 + \frac{\tilde{P}}{1 - \tilde{P}}\right) . \tag{10}$$

A comparison of (7)-(10) with theoretical relations (5), (6) reveals the following substantial differences.

1. The experimental exponent in the dependence $\tilde{\alpha} \approx \tilde{q}^m$ decreases from $m = 0.9$ at $\tilde{P} \to 0$ to $m = 0.6$ at $\tilde{P} \to 1$. The theory [2, 4] gives $m = 2/3$ for the entire range of reduced pressures.

2. The experimental dependence $\tilde{\alpha}(\tilde{P})$ [5] acquires the form $\tilde{\alpha} \approx \tilde{P}^{0.27}$ at $\tilde{P} \to 0$. According to theoretical relation (5) of [2] the heat transfer in the region of $\tilde{P} \to 0$ is practically independent of the pressure.

3. From generalizing formula (7) given in [5] it follows that for the entire range of reduced pressures the heat transfer depends on the characteristic dimension of the roughness as $\tilde{\alpha} \approx \tilde{l}^{0.133}$. Note that recent experimental studies [6, 7] give an even stronger dependence: $\tilde{\alpha} \approx (\tilde{l}^{0.2} - \tilde{l}^{0.3})$. The theoretical models [2, 4] do not provide an analysis of the effect of roughness on heat transfer in boiling.

The present work suggests an approoximate model of heat transfer in bubble boiling under natural-convection conditions that agrees with the results of experimental studies [5-7].

1. Bubble Boiling as Near-Wall Turbulence. In [8, 9] in an analysis of turbulent heat transfer in a two-phase bubbling layer the turbulent thermal diffusivity caused by bubble surface vibrations is introduced:

$$a_t \approx R^2 \omega ,$$

where ω is the frequency of natural vibrations of a bubble calculated by the Rayleigh formula

$$\omega \approx \sqrt{\left(\frac{\sigma}{\rho R^3}\right)} .$$

According to [8, 9], heat transfer in a bubble layer with the same radius R is determined by the effective thermal conductivity:

$$\lambda_* = \rho c_p (a + a_t) .$$

Hence the following expression for the specific heat flux is obtained:

$$q = - \rho c_p \left(a + k \sqrt{\left(\frac{\sigma R}{\rho}\right)} \right) \frac{dT}{dy} , \tag{11}$$

where k is a numerical constant equal approximately to unity.

Let us use relation (11) to describe the heat transfer in a two-phase near-wall layer existing on a heat-transfer surface under boiling conditions [2, 4]. Similarly to the Prandtl hypothesis of a linear dependence of the mixing path length on the transverse coordinate, we assume that for each value of the transverse coordinate y the largest contribution to turbulent heat transfer is made by a bubble of radius $R \approx y$. Then integrating (11) with respect to y at $q = \text{const}$, we obtain an expression for the temperature distribution in the near-wall boiling layer:

$$T = T_{\mathrm{w}} - \frac{2qa}{c_p \sigma} \left(\sqrt{\tilde{y}} - \ln \left(1 + \sqrt{\tilde{y}}\right) \right) . \tag{12}$$

Here, $\tilde{y} = k_1 \sigma y / (\rho a^2)$ is the dimensionless distance from the wall; $k_1 \approx k$ is a numerical constant.

At the outer boundary of the two-phase near-wall layer the liquid temperature must attain the saturation temperature:

$$y = \delta ; \quad T = T_{\mathrm{s}} .$$

Then from (12) an expression that relates the heat-transfer coefficient $\alpha = q/\Delta T$ to the dimensionless thickness of the near-wall liquid layer $\tilde{\delta} = \sigma \delta / (\rho a^2)$ follows:

$$1 = \frac{2 \alpha a}{c_p \sigma} \left(\sqrt{\tilde{\delta}} - \ln \left(1 + \sqrt{\tilde{\delta}}\right) \right) . \tag{13}$$

At $\tilde{\delta} \to 0$ from (13) a "laminar" formula for conductive heat transfer through the near-wall liquid film follows that is similar to the approach of [2]:

$$\alpha \approx \frac{\lambda}{\delta} .$$

At $\tilde{\delta} \to \infty$ relation (13) leads to a "turbulent" law of heat transfer [8, 9]:

$$\alpha^2 \approx \frac{\rho \sigma c_p^2}{\tilde{\delta}} .$$

Assuming that $\delta \approx \lambda / \alpha$ and evaluating the orders of the quantities ($k_1 \approx k \approx 1$; $\sigma \approx (10^{-2}-10^{-1})$ N/m; $\rho \approx 10^{-3}$ kg/m^3; $a \approx (10^{-7}-10^{-6})$ m^2/sec; $\alpha \approx (10^4-10^5)$ W/(m$^2 \cdot$K)), we obtain $\tilde{\delta} \approx 10^4-10^6$, $\tilde{\delta}^{1/2} \gg \ln(1 + \tilde{\delta}^{1/2})$. Hence it follows that the heat transfer through the near-wall boiling layer is practically always (with an accuracy of up to $\approx 5\%$) determined by the regularities of the "bubble-layer turbulence." As is known, in the Labuntsov approach [2] the thickness of the near-wall liquid film is calculated from the dynamic characteristics of the boiling process, namely, the radius and growth rate of a vapor bubble. In the present model the condition of dense packing of "macrobubbles" in the two-phase layer $\delta \approx L$ (Fig. 1) is assumed. This gives an expression for the heat-transfer coefficient in bubble boiling:

$$\alpha^2 \approx \frac{\rho \sigma c_p^2}{L}. \tag{14}$$

The next stage in the model is determination of the distance between vaporization centers or, which is the same, the surface density of vaporization centers

$$N \approx \frac{1}{L^2}. \tag{15}$$

2. Surface Density of Vaporization Centers. The first theoretical relation for N was suggested by Labuntsov [2]:

$$N \approx \frac{1}{R_*^2},$$

where the radius of a critical vapor nucleus R_* was determined from a linear approximation of the saturation curve:

$$R_* \approx \frac{\sigma T_s}{r \rho'' \Delta T}. \tag{16}$$

From a quadratic approximation of the saturation curve Yagov [4] obtained the following expression for the low-pressures region

$$R_* \approx \frac{\sigma T_s^2}{r \rho'' \Delta T^2}. \tag{17}$$

Relations (16), (17) lead to power-law dependences of the density of vaporization centers on the temperature drop: $N \approx \Delta T^4$ for the low-pressure region; $N \approx \Delta T^2$ for the high-pressure region. However, in a number of experimental studies (see, for instance, [6, 10]) a very strong dependence $N(\Delta T)$ was obtained over the entire pressure range (up to the thermodynamic critical point): $N \approx \Delta T^4 - \Delta T^5$. Hence it follows that the distance between vaporization centers is determined by some macroscale of the length of the bubble-boiling process rather than by the radius of a critical vapor nucleus R_*. In [11, 12] such a macroscale was determined from an analysis of the flow in a liquid film beneath vapor bubbles on a heating surface. As a result, for the low-pressure region $N \approx \Delta T^6$ was obtained, and for the high-pressure region $N \approx \Delta T^4$. In a recent experimental study [7] an even stronger dependence of the desity of vaporization centers on the temperature drop, namely, $N \approx \Delta T^7$, was established for a wide pressure range. To explain the indicated data, below use is made of a model of boiling-center formation based on the "quantum hypothesis."

For a critical nucleus of the vapor phase to appear in a liquid, energy equal to the free energy on its surface $E \approx \sigma R_*^2$ is needed [13]. Then in generation of critical vapor nuclei of radius R_* in the center of a cell of side L on the surface (Fig. 1) the energy $E \approx (\sigma R_*^2)(R_*/L)^2 \approx \sigma R_*^4/L^2$ will fall on each cell. The nucleation frequency of vapor nuclei can be evaluated from the time needed for a thermal wave to traverse the distance $R_*(\omega \approx a/R_*^2)$. We assume that the energy and frequency of nucleation of vapor nuclei are related in the same way as in a quantum oscillator [14]:

$$E \approx \hbar \omega, \tag{18}$$

where $\hbar$ is the Planck constant. Then the above considerations allow the following relation for the distance between vaporization centers to be written:

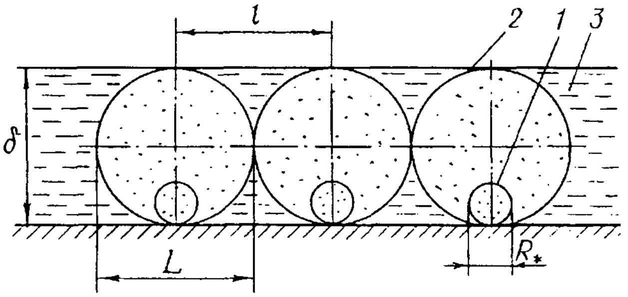

Fig. 1. Schematic of the near-wall picture of bubble boiling: 1) critical vapor nucleus; 2) macrobubble; 3) two-phase bubble layer.

$$L^2 \approx \frac{\sigma R_*^6}{a\hbar}\,. \tag{19}$$

Since application of the quantum hypothesis to describe the process of generation of vapor nuclei cannot be substantiated rigorously, below the well-known Motulevich "principle of relative correspondence" [15] is employed: "In determining relative quantities characterizing the deviation of the parameters in two phenomena close in their physical nature, considerably rougher models of the process can be used than in determining the absolute magnitudes with the same degree of accuracy."

Now we use the quantum hypothesis as a rough model to describe homogeneous nucleation in a superheated liquid volume, for which a strict theory exists [13]. Considering that a densely packed layer of critical vapor nuclei is formed in the liquid volume, from relation (18) we find the volume frequency of nucleation:

$$I \approx \omega/R_*^3 \approx E/(\hbar R_*^3) \approx (\sigma/R_*)/\hbar \approx \Delta P_*/\hbar\,.$$

The Laplace pressure drop across the surface of a thermodynamically equilibrium critical nucleus can be evaluated as the pressure drop of "limiting superheating curve – saturation curve" [13]

$$\Delta P_* = 2\sigma/R_* \approx \sigma^{3/2}/\sqrt{k_{\rm B}T_{\rm s}}\,,$$

where $k_{\rm B}$ is the Boltzmann constant.

Comparison of the quantities I obtained using the quantum hypothesis (the rough model) and the Kagan theoretical model of homogeneous nucleation [13] (the rigorous model) allows us to eliminate the Planck constant from relation (19):

$$\hbar \approx \mu L_*^3\,. \tag{20}$$

In the Kagan theory L_* is a quantity of the order of the intermolecular distance in the liquid. For the process of bubble boiling it is natural to use the surface roughness dimension as L_*: $L_* \approx l$. Thus, use of the "principle of relative correspondence" yields a relation for the surface density of vaporization centers that stems from (15), (19), (20):

$$N \approx \frac{1}{L^2} \approx \frac{a\mu l^3}{\sigma R_*^6}\,. \tag{21}$$

Note that the quantum hypothesis was used earlier in [16] to describe experimental data [7] for the surface density of vaporization centers.

3. **Limiting Relations for Heat Transfer.** Relations (14)-(17) and expression (21) make it possible to write resultant limiting relations for heat transfer in bubble boiling in the low- and high-pressure regions. These relations contain the Prandtl number for a liquid ($\Pr = \nu/a$) with small exponents so that it was assumed approximately that

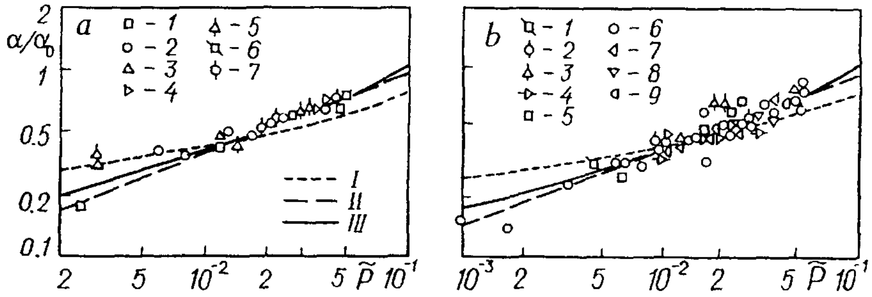

Fig. 2. Heat transfer in bubble boiling in the low-pressure region for cooling agents (a) and organic liquids (b) according to experimental data [5] and calculation by a relation of [11] (I) and by formula (22) of the present work (II) (III, approximation of the experimental data): a: 1) R-11, 2) R-12, 3) R-22, 4) R-23, 5) R-113, 6) R-114, 7) R-226; b: 1) CH_4, 2) C_2H_6, 3) C_4H_{10}, 4) C_3H_{12}, 5) CH_4O, 6) C_2H_6O, 7) C_3H_8O, 8) C_4H_{10}, 9) C_3H_6O.

$\mu \approx \rho a$. Simple evaluations show that here an error of any appreciable value can occur for two limiting cases with respect to the Prandtl number: liquid metals (Pr << 1) and highly viscous fluids (Pr >> 1), which are not considered in the present work. The resultant limiting relations for the heat-transfer coefficient are:

1. The low-pressure region ($\tilde{P} \to 0$)

$$\alpha_1 \approx \frac{q^{3/4} \lambda^{1/4} \rho''^{3/8} r^{3/8} l^{3/16}}{T_s^{3/4} \rho^{1/16} a^{1/8} \sigma^{5/16}} . \tag{22}$$

In comparing relation (22) and empirical relation (7) taken from [5], we take into account the important fact that in the region of $\tilde{P} \to 0$ only the vapor density depends on the pressure, which can be expressed in terms of the reduced pressure as

$$\rho''/\rho \approx \tilde{P} .$$

This allows us to represent relation (22) in dimensionless form as

$$\tilde{\alpha}_1 \approx A \tilde{P}^{3/8} \tilde{l}^{3/16} \tilde{q}^{3/4} . \tag{23}$$

Here A is a dimensionless constant that can be written, however, only in the form of a cumbersome formula that includes the liquid properties and the quantity scales prescribed by relation (8). To compare formula (22) with a file of experimental data [5], we carried out the procedure of selective numerical estimation described in [11]. As is seen from Fig. 2, there is good agreement between results calculated by formula (22) of the present work and experimental data [5] in the reduced-pressure range $0 \le \tilde{P} \le 0.1$.

2. The high-pressure region ($\tilde{P} \to 0$)

$$\alpha_2 \approx \frac{q^{3/5} \lambda^{2/5} \rho''^{3/5} r^{3/5} l^{3/10}}{T_s^{3/5} \rho^{1/10} a^{1/5} \sigma^{1/2}} . \tag{24}$$

A comparison of expression (24) with Yagov limiting relation (6) reveals virtual coincidence of the exponents of the physical quantities that depend strongly on the pressure: vapor density ρ'', saturation temperature T_s, specific heat of vaporization r, surface tension σ. In [4], based on extensive experimental data it was shown that formula (6) describes satisfactorily the heat transfer in the high-pressure region. Consequently, this must be valid to a large extent for formula (24). However, the latter, also includes a dependence, which is absent in [4], of the heat-transfer coefficient on the surface roughness dimension that satisfactorily fits recent experimental data [6, 7]. It is also of interest to note that the exponent in the dependence (24) of the heat-transfer coefficient on the reduced pressure yields the limiting value of empirical formula (9) taken from [5], which generalizes a large body of experimental data on boiling of organic liquids and cooling agents:

$$\tilde{P} = 1 \, , \quad m = 0.6 \, .$$

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NOTATION

λ, thermal conductivity of the liquid; a, thermal diffusivity of the liquid; μ, dynamic viscosity of the liquid; ν, kinematic viscosity of the liquid; ρ, liquid density; c_p, specific heat of the liquid; ρ'', vapor density; r, specific heat of vaporization; σ, surface tension; T, liquid temperature; δ, thickness of the near-wall boiling layer; q, specific heat flux; L, distance between vaporization centers; $\Delta T = T_w - T_s$, "wall–boiling liquid" temperature drop; α, heat-transfer coefficient; l, characteristic dimension of the surface roughness; y, transverse coordinate; R, radius of a vapor bubble; P, pressure; P_{cr}, pressure at the thermodynamic critical point; $\tilde{P} = P/P_{cr}$, reduced pressure; N, surface density of vaporization centers; E, energy of the quantum oscillator. Subscripts: w, wall; s, saturation; 0, scale quantity; 1, at $\tilde{P} \to 0$ (low pressures); 2, at $\tilde{P} \to 1$ (high pressures); t, turbulent; cr, critical; *, effective.

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